The Pairwise-Comparison Method
Lecture 11
Section 1.5

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Outline

The Method of Pairwise Comparisons

Definition (The Method of Pairwise Comparisons)

By the method of pairwise comparisons, each voter ranks the candidates. Then, for every pair (for every possible two-way race) of candidates,

- Determine which one was preferred more often.
- That candidate gets 1 point.
- If there is a tie, each candidate gets 1/2 point.

The candidate who gets the greatest number of points is the winner.

Then rank the candidates according to the number of points received.

Outline

Example

Example

Suppose that there are 3 candidates: A, B, C. The following table summarizes the voters' preferences.

	Preferences			
No. of voters	7	6	3	2
1st	A	B	B	C
2nd	B	A	C	B
3rd	C	C	A	A

- How many pairings are there?
- List the pairings.
- Count the votes for each pairing and determine the winner.

Example

Example

Suppose that there are 4 candidates: A, B, C, D. The following table summarizes the voters' preferences.

	Preferences			
No. of voters	11	8	7	4
1st	A	B	D	C
2nd	B	D	A	A
3rd	C	C	B	D
4th	D	A	C	B

- How many pairings are there?
- List the pairings.
- Count the votes for each pairing and determine the winner.

Example

Example

Suppose that there are 5 candidates: A, B, C, D, E. The following table summarizes the voters' preferences.

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	C	A	E	E
2nd	A	A	A	E	D	B	D
3rd	E	D	C	D	E	A	A
4th	D	C	E	B	B	D	B
5th	C	E	B	A	C	C	C

- How many pairings are there?
- List the pairings.
- Count the votes for each pairing and determine the winner.

Outline

The Number of Comparisons

- How many comparisons are there?
  - With 3 candidates, there are 3 comparisons.
  - With 5 candidates, there are 10 comparisons.

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  - With 6 candidates, how many comparisons would there be?

The Number of Comparisons

- How many comparisons are there?
  - With 3 candidates, there are 3 comparisons.
  - With 5 candidates, there are 10 comparisons.
  - With 6 candidates, how many comparisons would there be?
  - How many with 7 candidates?

Outline

- This method seems to take pretty much everything into account.
- So what could go wrong?

A Shortcoming

	Preferences						
No. of voters	6	4	4	4	2	1	1
1st	B	B	D	C	A	E	E
2nd	A	A	A	E	D	B	D
3rd	E	D	C	D	E	A	A
4th	D	C	E	B	B	D	B
5th	C	E	B	A	C	C	C

- Reconsider the previous example.

A Shortcoming

Example (A Shortcoming)

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	C	A	E	E
2nd	A	A	A	E	D	B	D
3rd	E	D	C	D	E	A	A
4th	D	C	E	B	B	D	B
5th	C	E	B	A	C	C	C

- At the last minute, candidate E drops out.

A Shortcoming

Example (A Shortcoming)

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	C	A	B	D
2nd	A	A	A	D	D	A	A
3rd	D	D	C	B	B	D	B
4th	C	C	B	A	C	C	C

- Now who is the winner?

A Shortcoming

Example (A Shortcoming)

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	C	A	B	D
2nd	A	A	A	D	D	A	A
3rd	D	D	C	B	B	D	B
4th	C	C	B	A	C	C	C

- Now who is the winner?
- Is that surprising?

The Perfect Voting Method

- Is there a voting method that has no shortcoming?

Outline

Assignment

Assignment

- Chapter 1: Exercises 41, 42, 43, 44, 45, 47, 49, 50.